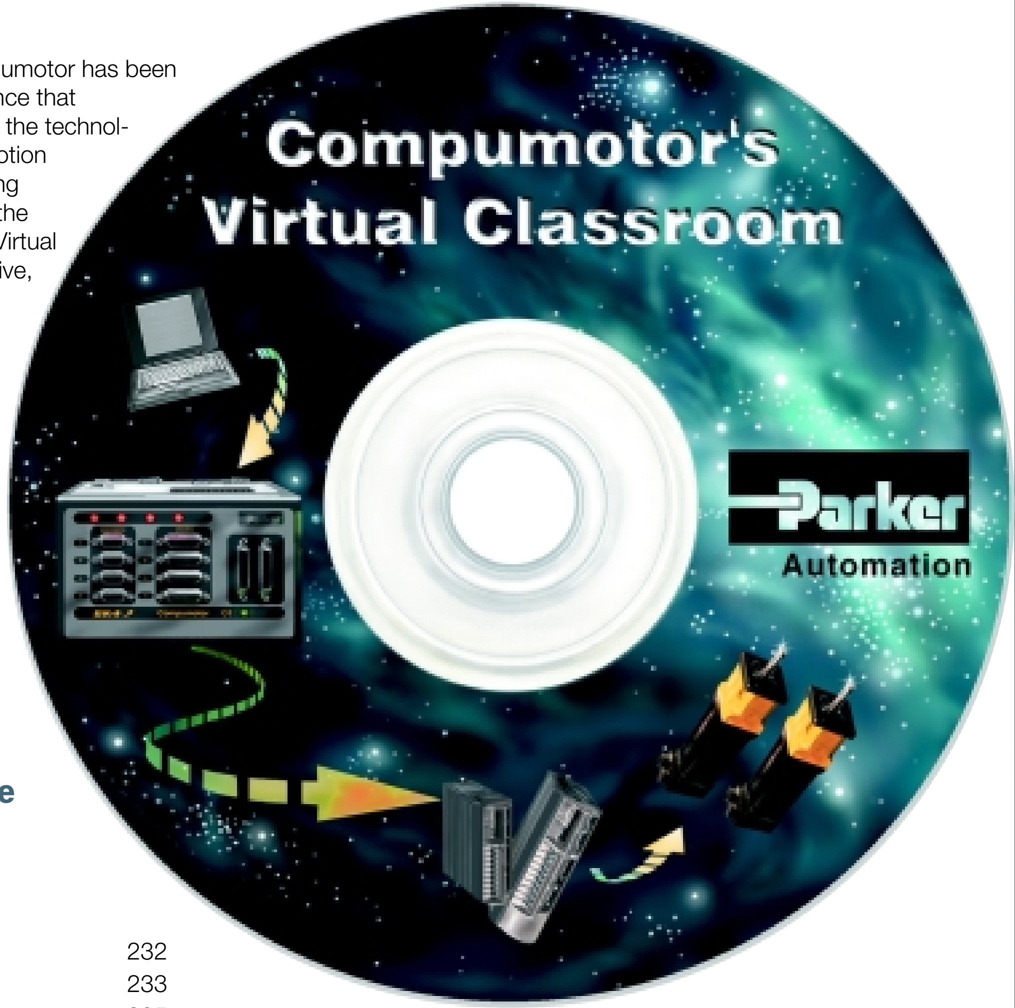
Engineering Reference Guide



Over the past twenty years, Compumotor has been developing an Engineering Reference that compiles important information on the technology and practical application of motion control. Compumotor's Engineering Reference was the foundation for the Virtual Classroom CD ROM. The Virtual Classroom CD-ROM is an interactive, multi-media tool that makes the Engineering Reference come alive. To request your free copy, go to www.compumotor.com.

For your convenience, we are providing an abbreviated version of the Engineering Reference that concentrates on the Sizing and Selection of a motion system. In the future, the complete Engineering Reference will be printed as a separate text to allow greater use and distribution to colleges and universities. Check the website for availability.

Engineering Reference Table of Contents:

Motor Sizing and Selection Process

Before you can select your motors drives and controls, you must define the mechanical system and determine the performance requirements of each axis of motion. The information in this section will help you:

1. Calculate move profile for each axis of motion.
2. Calculate total inertia for each axis of motion
3. Calculate torque required for acceleration for each axis of motion
4. Calculate max motor speed for each axis of motion
5. Review the application considerations to determine special

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Sizing Step 1: Move Profile

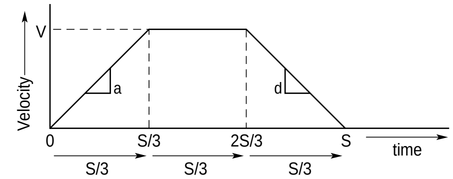
Before calculating torque requirements of an application, you need to know the velocities and accelerations needed. For those positioning applications where only a distance (X) and a time (S) to move that distance are known, the trapezoidal motion profile and formulas given below are a good starting point for determining your requirements. If velocity and acceleration parameters are already known, you can proceed to one of the specific application examples on the following pages.

Move distance X in time S.

Assume that:

1. Distance X/4 is moved in time S/3 (Acceleration)
2. Distance X/2 is moved in time S/3 (Run)
3. Distance X/4 is moved in time S/3 (Deceleration)

The graph would appear as follows:



The acceleration (a), velocity (v) and deceleration (d) may be calculated in terms of the knowns, X and S.

$$a = -d = \frac{2X}{t^2} = \frac{2\left(\frac{X}{4}\right)}{\left(\frac{S}{3}\right)^2} = \frac{\frac{X}{2} \times 9}{S^2} = \frac{4.5X}{S^2}$$

$$v = at = \frac{4.5X}{S^2} \times \frac{S}{3} = \frac{1.5X}{S}$$

Example

You need to move 6" in 2 seconds

$$a = -d = \frac{4.5\ (6\ \text{inches})}{(2\ \text{seconds})^2} = 6.75\ \frac{\text{inches}}{\text{second}^2}$$

$$v = \frac{1.5\ (6\ \text{inches})}{(2\ \text{seconds})} = 4.5\ \frac{\text{inches}}{\text{second}}$$

Common Move Profile Considerations

Distance: ______________ Inches of Travel ________________________ revolutions of motor

Move Time: ____ seconds

Accuracy: ____ arcminutes, degrees or inches

Repeatability: ____ arcseconds, degrees or inches

Duty Cycle

on tme: ____ seconds

off time: ____ seconds

Cycle Rate: ____ sec. min. hour

Sizing Step 2 and Step 3: Inertia and Torque Calculations

Leadscrew Drives

Leadscrews convert rotary motion to linear motion and come in a wide variety of configurations. Screws are available with different lengths, diameters, and thread pitches. Nuts range from the simple plastic variety to precision ground versions with recirculating ball bearings that can achieve very high accuracy.

The combination of microstepping and a quality leadscrew provides exceptional positioning resolution for many applications. A typical 10-pitch (10 threads per inch) screw attached to a 25,000 step/rev. motor provides a linear resolution of 0.000004" (4 millionths, or approximately 0.1 micron) per step. A flexible coupling should be used between the leadscrew and the motor to provide some damping. The coupling will also prevent excessive motor bearing loading due to any misalignment.

Leadscrew Application Data

Inertia of Leadscrews per Inch

Diameter In.	Steel	Brass	Alum.	
0.25	0.0017	0.0018	0.0006	oz-in^2
0.50	0.0275	0.0295	0.0094	oz-in^2
0.75	0.1392	0.1491	0.0478	oz-in^2
1.00	0.4398	0.4712	0.1512	oz-in^2
1.25	1.0738	1.1505	0.3691	oz-in^2
1.50	2.2266	2.3857	0.7654	oz-in^2
1.75	4.1251	4.4197	1.4180	oz-in^2
2.00	7.0372	7.5399	2.4190	oz-in^2
2.25	11.2723	12.0774	3.8748	oz-in^2
2.50	17.1807	18.4079	5.9059	oz-in^2

Diameter In.	Steel	Brass	Alum.	
2.75	25.1543	26.9510	8.6468	oz-in^2
3.00	35.6259	38.1707	12.2464	oz-in^2
3.25	49.0699	52.5749	16.8678	oz-in^2
3.50	66.0015	70.7159	22.6880	oz-in^2
3.75	86.9774	93.1901	29.8985	oz-in^2
4.00	112.5956	120.6381	38.7047	oz-in^2
4.25	143.4951	153.7448	49.3264	oz-in^2
4.50	180.3564	193.2390	61.9975	oz-in^2
4.75	223.9009	239.8939	76.9659	oz-in^2
5.00	274.8916	294.5267	94.4940	oz-in^2

Coefficients of Static Friction Materials

(Dry Contact Unless Noted)	μ_S
Steel on Steel	0.58
Steel on Steel (lubricated)	0.15
Aluminum on Steel	0.45
Copper on Steel	0.22
Brass on Steel	0.19
PTFE on Steel	0.04

Leadscrew Efficiencies Type	Efficiency (%) High	Median	Low
Ball-nut	95	90	85
Acme with metal nut*	55	40	35
Acme with plastic nut	85	65	50

*Since metallic nuts usually require a viscous lubricant, the coefficient of friction is both speed and temperature dependent.

Leadscrew Drives

Vertical or Horizontal Application:

ST	Screw type, ball or acme	ST = ________
e	Efficiency of screw	e = ________ %
μ_S	Friction coefficient	μ_S = ________
L	Length ofscrew	L = ________ inches
D	Diameter of screw	D = ________ inches
p	Pitch	p = ________ threads/inch
W	Weight of load	W = ________ lbs.
F	Breakaway force	F = ________ ounces
Directly coupled to the motor?		yes/no ________
If yes, CT – Coupling type		________
If no, belt & pulley or gears		________
Radius of pulley or gear		________ inches
Gear: Number of teeth – Gear 1		________
Number of teeth – Gear 2		________
Weight of pulley or gear		________ ounces
Weight of belt		________ ounces

Leadscrew Formulas

The torque required to drive load W using a leadscrew with pitch (p) and efficiency (e) has the following components:

$$T_{Total} = T_{Friction} + T_{Acceleration}$$

$$T_{Friction} = \frac{F}{2\pi pe}$$

Where:
F = frictional force in ounces
p = pitch in revs/in
e = leadscrew efficiency

$F = \mu_s W$ for horizontal surfaces where μ_s = coefficient of static friction and W is the weight of the load. This friction component is often called "breakaway".

Dynamic Friction: $F = \mu_D W$ is the coefficient to use for friction during a move profile. However, torque calculations for acceleration should use the worst case friction coefficient, μ_s.

$$T_{Accel} = \frac{1}{g} \left(J_{Load} + J_{Leadscrew} + J_{Motor}\right) \frac{\omega}{t}$$

$$\omega = 2\pi pv$$

$$J_{Load} = \frac{W}{(2\pi p)^2} ; J_{Leadscrew} = \frac{\pi L \rho R^4}{2}$$

Where:
T = torque, oz-in
ω = angular velocity, radians/sec
t = time, seconds
v = linear velocity, in/sec
L = length, inches
R = radius, inches
ρ = density, ounces/in^3
g = gravity constant, 386 in/sec^2

The formula for load inertia converts linear inertia into the rotational equivalent as reflected to the motor shaft by the leadscrew.

Problem

Find the torque required to accelerate a 200-lb steel load sliding on a steel table to 2 inches per second in 100 milliseconds using a 5 thread/inch steel leadscrew 36 inches long and 1.5 inches in diameter. Assume that the leadscrew has an Acme thread and uses a plastic nut. Motor inertia is given as 6.56 oz-in^2. In this example, we assume a horizontally oriented leadscrew where the force of gravity is perpendicular to the direction of motion. In non-horizontal orientations, leadscrews will transmit varying degrees of influence from gravity to the motor, depending on the angle of inclination. Compumotor Sizing Software automatically calculates these torques using vector analysis.

1. Calculate the torque required to overcome friction. The coefficient of static friction for steel-to-steel lubricant contact is 0.15. The median value of efficiency for an Acme thread and plastic nut is 0.65. Therefore:

$$F = \mu_s W = 0.15\,(200\text{ lb})\ \frac{(16\text{ oz})}{\text{lb}} = 480\text{ oz}$$

$$T_{Friction} = \frac{F}{2\pi pe} = \frac{480\text{ oz}}{\frac{2\pi}{\text{rev}} \times \frac{5\text{ rev}}{\text{in}} \times 0.65} = 23.51\text{ oz-in}$$

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2. Compute the rotational inertia of the load and the rotational inertia of the leadscrew:

$$J_{Load} = \frac{W}{(2\pi p)^2} = \frac{200\text{ lb}}{\frac{(2\pi 5)^2}{\text{in}^2}} \times \frac{16\text{ oz}}{\text{lb}} = 3.24\text{ oz-in}^2$$

$$J_{Leadscrew} = \frac{\pi L\rho R^4}{2} = \frac{\pi}{2}(36\text{ in})(4.48\ \frac{\text{oz}}{\text{in}^3})(0.75\text{ in})^4 = 80.16\text{ oz-in}^2$$

3. The torque required to accelerate the load may now be computed since the motor inertia was given:

$$T_{Accel} = \frac{1}{g}\left(\frac{1}{e}J_{Load} + J_{Leadscrew} + J_{Motor}\right)\frac{\omega}{t}$$

$$\omega = 2\pi\left(\frac{5}{\text{in}}\right)\left(\frac{2\text{ in}}{\text{sec}}\right) = \frac{20\pi}{\text{sec}}$$

$$= \frac{1}{386\text{ in/sec}^2}[4.99 + 80.16 + 6.56(\text{oz-in}^2)]\ \frac{\frac{20\pi}{\text{sec}}}{0.1\text{sec}}$$

$$= 149\text{ oz-in}$$

$$T_{Total} = T_{Friction} + T_{Accel}$$

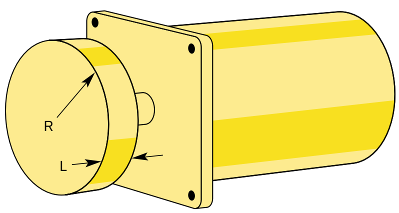
$$T_{Total} = 23.51\text{ oz-in} + 149\text{ oz-in} = 172.51\text{ oz-in}$$

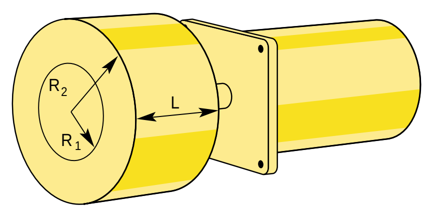
Directly Driven Loads

There are many applications where the motion being controlled is rotary and the low-speed smoothness and high resolution of a Compumotor system can be used to eliminate gear trains or other mechanical linkages. In direct drive applications, a motor is typically connected to the load through a flexible or compliant coupling. This coupling provides a small amount of damping and helps correct for any mechanical misalignment.

Direct drive is attractive when mechanical simplicity is desirable and the load being driven is of moderate inertia.

Direct Drive Formulas





R – Radius
R(1) – Inner radius
R(2) – Outer radius
L – Length
W – Weight of disc
ρ – Density/Material
g – Gravity constant

R = ______________________ inches
R(1) = ______________________ inches
R(2) = ______________________ inches
L = ______________________ inches
W = ______________________ ounces
ρ = ______________________ ounces/inch³
g = 386 in/sec²

Solid Cylinder (oz-in²)

Inertia: $J_{Load} = \frac{WR^2}{2}$

Where weight and radius are known

Inertia (oz-in²) $J_{Load} = \frac{\pi L\rho R^4}{2}$

Where ρ, the material density is known

Weight $W = \pi L\rho R^2$

Inertia may be calculated knowing either the weight and radius of the solid cylinder (W and R) or its density, radius and length (ρ, R and L.)

The torque required to accelerate any load is:

T (oz-in) = Ja

$a = \frac{\omega_2 - \omega_1}{t}$ = 2π (accel.) for Accel. in rps²

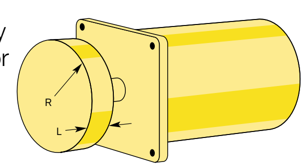
Where:

- a = angular acceleration, radians/sec²
- ω_2 = final velocity, radians/sec
- ω_1 = initial velocity, radians/sec
- t = time for velocity change, seconds
- J = inertia in units of oz-in²

The angular acceleration equals the time rate of change of the angular velocity. For loads accelerated from zero, $\omega_1 = 0$ and $a = \frac{\omega}{t}$

$$T_{Total} = \frac{1}{g}(J_{Load} + J_{Motor})\frac{\omega}{t}$$

T_{Total} represents the torque the motor must deliver. The gravity constant (g) in the denominator represents acceleration due to gravity (386 in/sec²) and converts inertia from units of oz-in² to oz-in-sec².



Hollow Cylinder

$$J_{Load} = \frac{W}{2}(R^2_1 + R^2_2)$$

Where W, the weight, is known

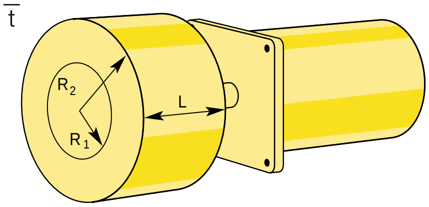
or

$$J_{Load} = \frac{\pi L \rho}{2}(R^4_2 - R^4_1)$$

Where r, the density, is known

$$W = \pi L \rho (R^2_2 - R^2_1)$$

$$T = \frac{1}{g}(J_{Load} + J_{Motor})\frac{\omega}{t}$$



Problem

Calculate the motor torque required to accelerate a solid cylinder of aluminum 5" in radius and 0.25" thick from rest to 2.1 radians/sec (0.33 revs/sec) in 0.25 seconds. First, calculate J_{Load} using the density for aluminum of 1.54 oz/in^3.

$$J_{Load} = \frac{\pi L \rho R^4}{2} = \frac{\pi \times 0.25 \times 1.54 \times 5^4}{2} = 378 \text{ oz-in}^2$$

Assume the rotor inertia of the motor you will use is 37.8 oz-in^2.

$$T_{Total} = \frac{1}{g}(J_{Load} + J_{Motor}) \times \frac{\omega}{t}$$

$$= \frac{1}{386} \times (378 + 37.8) \times \frac{2.1}{0.25}$$

$$= 9.05 \text{ oz-in}$$

Gear Drives

Traditional gear drives are more commonly used with step motors. The fine resolution of a microstepping motor can make gearing unnecessary in many applications. Gears generally have undesirable efficiency, wear characteristics, backlash, and can be noisy.

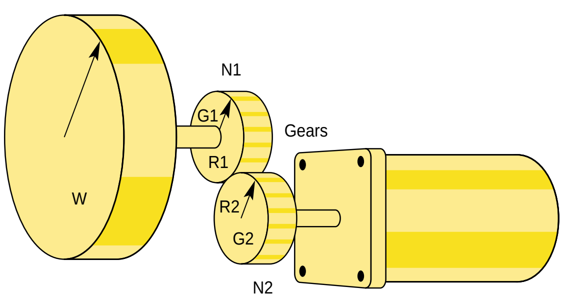
Gears are useful, however, when very large inertias must be moved because the inertia of the load reflected back to the motor through the gearing is divided by the square of the gear ratio.

In this manner, large inertial loads can be moved while maintaining a good load-inertia to rotor-inertia ratio (less than 10:1).

Gear Driven Loads

R – Radius
R(1) – Radius gear #1
R(2) – Radius gear #2
N(1) – Number of teeth G#1
N(2) – Number of teeth G#2
G – Gear ratio $\frac{N(1)}{N(2)}$
W – Weight of load
W(1) – Weight G#1
W(2) – Weight G#2
L – Length L =
F – Friction F =
BT – Breakaway torque

R = ________ inches
R(1) = ________ inches
R(2) = ________ inches
N(1) = ________
N(2) = ________
G = ________
W = ________ ounces
W(1) = ________ ounces
W(2) = ________ ounces
inches________
____
BT = ________ ounce/inches



Gear Drive Formulas

$$J_{Load} = \frac{W_{Load}}{2} R^2_{Load} \left(\frac{N_{Gear\,2}}{N_{Gear\,1}}\right)^2$$

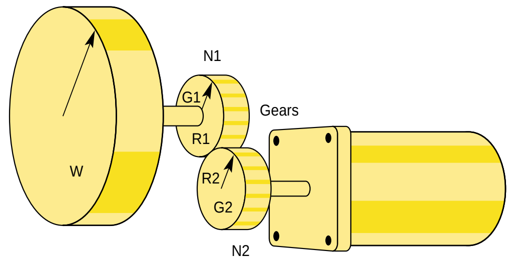
or

$$J_{Load} = \frac{\pi L_{Load} \rho_{Load}}{2} R^4_{Load} \left(\frac{N_{Gear\,2}}{N_{Gear\,1}}\right)^2$$

$$J_{Gear1} = \frac{W_{Gear1}}{2} R^2_{Gear1} \left(\frac{N_{Gear\,2}}{N_{Gear\,1}}\right)^2$$

$$J_{Gear2} = \frac{W_{Gear2}}{2} R^2_{Gear2}$$

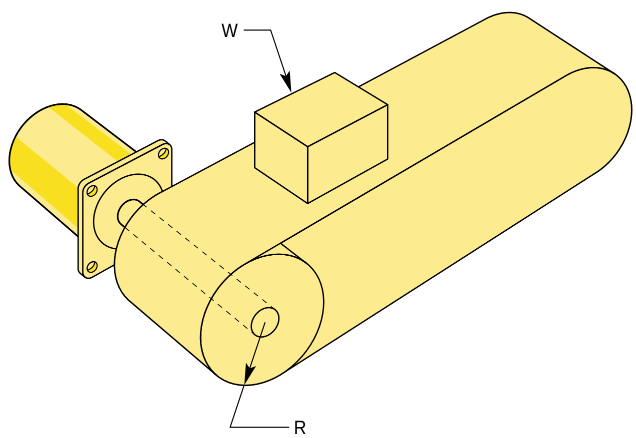
$$T_{Total} = \frac{1}{g} (J_{Load} + J_{Gear1} + J_{Gear2} + J_{Motor}) \frac{\omega}{t}$$



Where:

- J = inertia, oz-in ($gm\text{-}cm^2$) "as seen by the motor"
- T = torque, oz-in (gm-cm)
- W = weight, oz (gm)
- R = radius, in. (cm)
- N = number of gear teeth (constant)
- L = length, in (cm)
- ρ = density, oz/in^3 (gm/cm^3)
- ω = angular velocity, radians/sec @ motor shaft
- t = time, seconds
- g = gravity constant, 386 in/sec^2

Tangential Drives



R– Radius	R = ______________________ inches
W – Weight (include weight of belt or chain)	W = ______________________ ounces
W(P) – Weight of pulley or material	W(P) ______________________ ounces
F – Breakaway force	F = ______________________ ounces
V – Linear velocity	V = ______________________ inches/sec
CT – Coupling type	CT = ______________________
SL – Side load	SL = ______________________

Tangential Drive Formulas

$$T_{Total} = T_{Load} + T_{Pulley} + T_{Belt} + T_{Motor} + T_{Friction}$$

$$T_{Total} = \frac{1}{g}(J_{Load} + J_{Pulley} + J_{Belt} + J_{Motor})\frac{\omega}{t} + T_{Friction}$$

$$J_{Load} = W_L R^2$$

$$J_{Pulley} = \frac{W_p R^2}{2}$$

Remember to multiply by 2 if there are two pulleys.

$$J_{Belt} = W_B R^2$$

$$T_{Friction} = FR$$

$$\omega = \frac{V}{R}$$

Where:

- T = torque, oz-in (gm-cm)
- ω = angular velocity, radians/sec
- t = time, seconds
- W_L = weight of the load, oz
- W_P = pulley weight, oz
- W_B = belt or rack weight, oz
- F = frictional force, oz (gm)
- R = radius, in (cm)
- V = linear velocity
- g = gravity constant, 386 in/sec²
- ρ = density, oz/in³

Problem

What torque is required to accelerate a 5-lb load to a velocity of 20 inches per second in 10 milliseconds using a flat timing belt? The motor drives a 2-inch diameter steel pulley 1/2-inch wide. The timing belt weighs 12 oz. Load static friction is 30 ozs. Motor rotor inertia is 10.24 oz-in.²

$$J_{Load} = W_L R^2 = 5\text{ lb} \times 16\,\frac{\text{oz}}{\text{lb}} \times (1\text{ in})^2 = 80\text{ oz-in}^2$$

$$J_{Pulley} = \frac{2(\pi L \rho R^4)}{2} = \pi \times 0.5\text{ in} \times (4.48\text{ oz/in}^3)(1\text{ in})^4$$

$$= 7.04\text{ oz-in}^2$$

$$J_{Belt} = W_B R^2 = 12\text{ oz}(1\text{ in})^2 = 12\text{ oz-in}^2$$

$$T_{Friction} = F \times R = 30\text{ oz} \times 1\text{ in} = 30\text{ oz-in}$$

$$\omega = \frac{V}{R} = 20\,\frac{\text{in}}{\text{sec}} \times \frac{1\text{ rad}}{1\text{ in}} = 20\,\frac{\text{rad}}{\text{sec}}$$

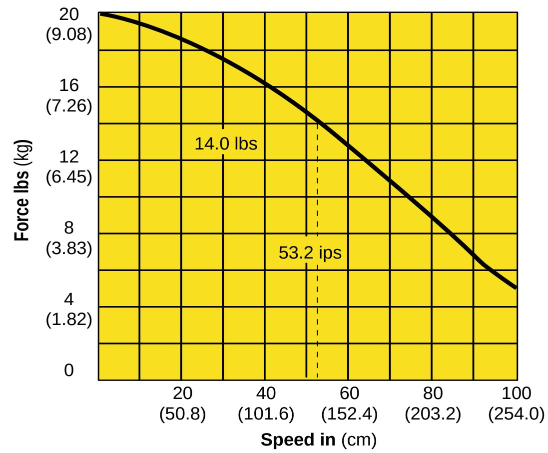
$$T_{Total} = \frac{1}{386}(80 + 7.04 + 12 + 10.24)\frac{20}{.01} + 30$$

$$T_{Total} = 596.2\text{ oz-in}$$

Sizing Step 4: Motor/Drive Selection

Based on Continuous Torque Requirements

Having calculated the torque requirements for an application, you can select the motor/drive suited to your needs. Microstepping motor systems (Gemini, ZETA Series OEM750 Series) have speed/torque curves based on continuous duty operation. To choose a motor, simply plot total torque vs. velocity on the speed/torque curve. This point should fall under the curve and allow approximately a 50% margin for safety. A ZETA106-178 and a ZETA83-135 curve are shown here. *Note: When selecting a ZETA or Gemini product, a 50% torque margin is not required.*



Example

Assume the following results from load calculations:

F =	25 oz-in	Friction torque
A =	175 oz-in	Acceleration torque
T =	200 oz-in	Total torque
V =	15 rev/sec	Maximum velocity

The ZETA83-135 has approximately 250 oz-in available at V max (25% more than required). The Zeta106-178 has 375 oz-in available, an 88% margin.

In this case, we would select the Zeta106-178 motor/drive to assure a sufficient torque margin to allow for changing load conditions.

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